

Multiparty Sessions based on Proof Nets

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1 Introduction

Since their inception, sessions [12, 19] and multiparty sessions [11] have been gaining momentum as a very useful foundation for the description and verification of *structured interactions*. Interestingly, recent works have established a close correspondence between typed, synchronous pi-calculus processes and sequent proofs of a variation of Intuitionistic Linear Logic [3]. This particular interpretation of Linear proofs is considered a sessions system because it has (for practical purposes) the same type constructors but with a clear logical motivation. In this paper we outline a system based on an interpretation of the proof objects of Classical Linear Logic, namely Proof Nets [8], improving our previous work [16]. The process language resembles Solos [14] and exhibits asynchrony in both input and output. Proof Nets have a number of advantages over sequent proofs, such as increased potential for parallelism and a very appealing graphical notation that could be seen as a new kind of *global type* [11]. Nevertheless, accurate logical interpretations are typically deterministic, which limits their applicability to concurrent programming. However, with a very modest adaptation that enables *synchronisation*, non-deterministic behaviours can be allowed without compromising the basic properties of interest, namely strong normalisation and deadlock-freedom.

Let us distinguish *multiparty behaviours* and the *multiparty session types* (global types) of [11]. A multiparty behaviour emerges when more than two processes can be part of the same session, and this is achieved at the operational level by a synchronisation mechanism such as the multicast request $\bar{a}[2..n](\vec{s}).P$ [11]. We propose a similar mechanism in the form of replications with synchronisation, $!a_1(x_1) \cdots a_n(x_n).P$, which allow a service to be activated with multiple parties. A global type captures the interactions and sequencing constraints of the complete protocol of a program. In our proposal, the equivalent to a global type is the *proof net* of the program. Although our approach is technically very different, we believe that the logical foundations and simpler meta-theory are appealing. We show how pi-calculus channels with *i/o* type and a multi-party interaction from [11] can be encoded.

2 The Process Interpretation

Syntax The language is inspired by proof nets except that connectives have explicit locations (names). Types are ranged over by A, B, C , with type variables ranging over X, Y . We assume a countable set of *names*, ranged over by a, b, c, x, y, z, r, k . Then, \vec{b} stands for a sequence b_1, \dots, b_n of length $|\vec{b}| = n$, and similarly for types. Processes, P, Q, R , are defined as follows:

$$\begin{array}{lclclclclclclclcl}
 P & ::= & \bar{a}(\tilde{A}, \tilde{x}) & \mid & a(\tilde{X}, \tilde{y}) & \mid & \bar{a} \triangleleft \mathbf{i}(b) & \mid & a \triangleright \{\mathbf{i}(x_i:A_i).P_i\}_{i \in I} & \mid & ?\bar{a}(b) & \mid & !a_i(x_i:A_i)_{i \in I}.P \\
 & & ab & \mid & X \mapsto A & \mid & (\nu a:A)P & \mid & (\nu X)P & \mid & (P \mid Q) & \mid & \mathbf{0}
 \end{array}$$

There are two kinds of *solos*-like [14] communication devices: $a(\tilde{X}, \tilde{y})$ and $\bar{a}(\tilde{A}, \tilde{x})$. In typed processes, we will be using the nullary signals a for $a()$ and \bar{a} for $\bar{a}()$,¹ the binary input $a(b, c)$ (resp.

¹They have no computational content, but without them reduction leaves garbage axioms of unit type.

output $\bar{a}(b, c)$) and the *asynchronous* polymorphic input $a(X, b)$ (resp. output $\bar{a}(B, b)$). The explicit substitution, ab , interprets Linear Logic axioms; this is standard in related works [16, 2]. The type alias $X \mapsto A$ is simply a typing device, and the scope of X is restricted with $(\nu X)P$. The branching connective $a \triangleright \{i(x_i:A_i).P_i\}_{i \in I}$, with $I = \{1, \dots, n\}$, written also in the form $a \triangleright \{1(x_1:A_1).P_1 \parallel \dots \parallel n(x_n:A_n).P_n\}$, offers an indexed sequence of alternative behaviours. One of these can be selected using $\bar{a} \triangleleft k(b)$ with $k \in I$. Our notion of replication enables synchronisation, similarly to a multiparty “accept” (cf. [11]). The notation is $!a_i(x_i:A_i)_{i \in I}.P$, written also as $!a_1(x_1:A_1) \cdots a_n(x_n:A_n).P$ with $n \geq 1$. Dually, $? \bar{a}(b)$ can be thought as a “request.”

Free, passive, and active names The *free names* ($\text{fn}(P)$) are defined in the standard way. We just note that the only bound names are a in $(\nu a:A)P$ and the x_i in $a \triangleright \{i(x_i:A_i).P_i\}_{i \in I}$ and $!a_i(x_i:A_i)_{i \in I}.P$. The *passive names* ($\text{pn}(P)$) are defined similarly to $\text{fn}(P)$ except for:

$$\begin{aligned} \text{pn}(a(\tilde{X}, \tilde{x})) &= \text{pn}(\bar{a}(\tilde{A}, \tilde{x})) = \{\tilde{x}\} & \text{pn}(\bar{a} \triangleleft i(b)) &= \text{pn}(!\bar{a}(b)) = \{b\} & \text{pn}(ab) &= \emptyset \\ \text{pn}(a \triangleright \{i(x_i:A_i).P_i\}_{i \in I}) &= \cup_{i \in I} (\text{pn}(P_i) \setminus \{x_i\}) & \text{pn}(!a_i(x_i:A_i)_{i \in I}.P) &= \text{pn}(P) \setminus \cup_{i \in I} \{x_i\} \end{aligned}$$

The *active names* ($\text{an}(P)$) are defined by $\text{an}(P) = \text{fn}(P) \setminus \text{pn}(P)$. For example, y in $(\nu x)(a(x, y) \mid xy)$ is not active. (As usual, we assume the name convention.)

Structure Equivalence With \equiv we denote the least congruence on processes that is an equivalence relation, equates processes up to α -conversion, satisfies the abelian monoid laws for parallel composition, the usual laws for scope extrusion, and satisfies the following axioms:²

$$\begin{aligned} (\nu X)\mathbf{0} &\equiv \mathbf{0} & (\nu X)P \mid Q &\equiv (\nu X)(P \mid Q) & (X \notin \text{ftv}(Q)) \\ (\nu X)(\nu Y)P &\equiv (\nu Y)(\nu X)P & (\nu X)(\nu a:A)P &\equiv (\nu a:A)(\nu X)P & (X \notin \text{ftv}(A)) \\ ab &\equiv ba & ab \mid !a_1(x_1:A_1) \cdots a(x:A) \cdots a_n(x_n:A_n).P &\equiv ab \mid !a_1(x_1:A_1) \cdots b(x:A) \cdots a_n(x_n:A_n).P \end{aligned}$$

The most notable axiom is the last one, which effects a forwarding, *e.g.*, $? \bar{a}(x) \mid ab \mid !b(y).P \equiv ab \mid ? \bar{a}(x) \mid !a(y).P$. As can be seen next, the term on the right can now reduce.

Reduction “ \longrightarrow ” is the smallest binary relation on terms such that:

$$\begin{aligned} \bar{a}(\tilde{A}, \tilde{y}) \mid a(\tilde{X}, \tilde{x}) &\longrightarrow \widetilde{X \mapsto A} \mid \tilde{x}\tilde{y} & |\tilde{A}| &= |\tilde{X}|, |\tilde{x}| = |\tilde{y}| & \text{(R-Com)} \\ \bar{a} \triangleleft k(b) \mid a \triangleright \{i(x_i:A_i).P_i\}_{i \in I} &\longrightarrow P_k\{b/x_k\} & k &\in I & \text{(R-Sel)} \\ \prod_{i \in I} ? \bar{a}_i(b_i) \mid !a_i(x_i:A_i)_{i \in I}.P &\longrightarrow P\{b_i/x_i\}_{i \in I} \mid !a_i(x_i:A_i)_{i \in I}.P & & & \text{(R-Sync)} \\ (\nu a:A)(ab \mid P) &\longrightarrow P\{b/a\} & a &\neq b, a \in \text{an}(P) & \text{(R-Ax)} \\ P \equiv P' &\longrightarrow Q' \equiv Q \Rightarrow P \longrightarrow Q & \text{(R-Str)} & & P \longrightarrow Q \Rightarrow C[P] \longrightarrow C[Q] & \text{(R-Ctx)} \\ \text{Contexts in (R-Ctx):} & C[\cdot] ::= \cdot \mid (C[\cdot] \mid P) \mid (\nu a:A)C[\cdot] \mid (\nu X)C[\cdot] \end{aligned}$$

(R-Com) resembles solos reduction [14] but with explicit fusions [7, 16]. Specifically, given two vectors \tilde{x} and \tilde{y} of length n , the notation $\tilde{x}\tilde{y}$ stands for $x_1y_1 \mid \dots \mid x_ny_n$ or $\mathbf{0}$ if the vectors are empty. For polymorphism we create type aliases: $\widetilde{X \mapsto A}$ stands for $X_1 \mapsto A_1 \mid \dots \mid X_n \mapsto A_n$. Combined type and name communication appears also in a synchronous setting [18]. (R-Sel) is standard. In (R-Sync) we synchronise on all a_i , obtaining a form of multi-party session against $? \bar{a}_1(b_1) \mid \dots \mid ? \bar{a}_n(b_n)$. (R-Ax) effects a capture-avoiding name substitution, defined in the standard way. The side-condition $a \neq b$ guarantees that no bound name becomes free; $a \in \text{an}(P)$ ensures that the cut is applied correctly, that is, against two (or more) conclusions.

²The free type variables ($\text{ftv}(P)$) are defined in a standard way, noting that $\text{ftv}((\nu X)P) = \text{ftv}(P) \setminus X$. The free type variables of a type A ($\text{ftv}(A)$) are also standard and arise from \forall/\exists .

The Caires-Pfenning axiom reduction (R-Ax) is based on $(\nu a)([a \leftrightarrow b] \mid P) \longrightarrow P\{b/a\}(a \neq b)$ from [17], which is similar to the “Cleanup” rule of [1]. However, in an asynchronous language, this rule breaks subject reduction, which motivates our side-condition $a \in \text{an}(P)$. For example, $Q \doteq (\nu x, y)(\bar{a}\langle x, y \rangle \mid [x \leftrightarrow b] \mid [y \leftrightarrow c])$ is typable in the system of [5], with conclusion $b: A, c: B \vdash Q :: a: A \otimes B$, but it reduces to $(\nu y)(\bar{a}\langle b, y \rangle \mid [y \leftrightarrow c])$ which is not typable.³

Types and duality The types, ranged over by A, B, C, D, \dots , are linear logic formulae [8]:

$$A ::= \mathbf{1} \mid \perp \mid A \otimes B \mid A \wp B \mid A \& B \mid A \oplus B \mid !_{\mathbf{m}} A \mid ?_{\mathbf{m}} A \mid \forall X. A \mid \exists X. A \mid X \mid \sim X$$

The *mode* \mathbf{m} can be ε (empty) or \star (synchronising): ε is a formality and is never shown; \star is used to enforce some restrictions, but does not generally alter the meaning of types. Negation $\sim(\cdot)$, which corresponds to duality, is an involution on types ($\sim(\sim A) = A$) defined in the usual way (we use the notation from [10]):

$$\begin{aligned} \sim \mathbf{1} &\doteq \perp & \sim \perp &\doteq \mathbf{1} & \sim(A \otimes B) &\doteq \sim A \wp \sim B & \sim(A \wp B) &\doteq \sim A \otimes \sim B \\ \sim(A \& B) &\doteq \sim A \oplus \sim B & \sim(A \oplus B) &\doteq \sim A \& \sim B & \sim(!_m A) &\doteq ?_m \sim A & \sim(?_m A) &\doteq !_m \sim A \\ \sim(\forall X. A) &\doteq \exists X. \sim A & \sim(\exists X. A) &\doteq \forall X. \sim A & \sim(\sim X) &\doteq X \end{aligned}$$

The multiplicative conjunction $A \otimes B$ (with unit $\mathbf{1}$) is the type of a channel that communicates a name of type A and a name of type B , offered by disconnected terms; it can be thought as an “output.” The multiplicative disjunction $A \wp B$ (with unit \perp) is only different in that the communicated names can be offered by one term; this possibility of dependency makes it an “input.” In a standard way, the additive conjunction $A \& B$ is an external choice (branching), and dually additive disjunction $A \oplus B$ is an internal choice (selection). Ignoring modes, the exponential types $!A$ and $?A$ can be understood as a decomposition of the “shared” type in sessions: $!A$ is assigned to a persistent term that offers A ; dually, $?A$ can be assigned to any name with type A so that it can communicate with $! \sim A$. The second-order types $\forall X. A$ and $\exists X. A$ are standard, as is type substitution: $A[B/X]$ stands for A with B for X , and $\sim B$ for $\sim X$.

Judgements and interfaces A judgement $P \triangleright \Gamma$ denotes that term P can be assigned the *interface* Γ . Interfaces, ranging over Γ, Δ , are sequences with possible repetition, defined by:

$$\Gamma ::= \emptyset \mid \Gamma, a: A \mid \Gamma, [a: A] \mid \Gamma, X$$

$a: A$ is standard. A *discharged occurrence* $[a: A]$ indicates that a has been used as A : it serves to protect linearity, since a can no longer be used. Γ, X records that X appears free in the term, ensuring freshness of type variables. $\tilde{a}: \tilde{A}$ stands for $a_1: A_1, \dots, a_n: A_n$. $? \Gamma$ stands for $\tilde{a}: ? \tilde{A}$, i.e., $a_1: ?A_1, \dots, a_n: ?A_n$. Similarly, $[\tilde{a}: \tilde{A}]$ means $[\tilde{a}: \tilde{A}]$. Let $\text{fn}(a: A) = \text{fn}([a: A]) = a$ and $\text{fn}(X) = \emptyset$, plus the obvious definition for free type variables ($\text{ftv}(\Gamma)$). We consider *well-formed interfaces*, in which only $a: ?A$ can appear multiple times, but $a: ?_{\star} A$ cannot. Moreover, (Γ, X) is well-formed when Γ is well-formed and $\exists \sigma. \sigma(\Gamma) = [\Delta], \Sigma$ such that $X \notin \text{ftv}(\Sigma)$. For example in the (\forall) rule the conclusion is $\Gamma, [x: A], X, a: \forall X. A$ with X possibly free in A .

Subtyping The usual structural rules of Linear Logic are incorporated into the relation $\Gamma \preceq \Delta$:

$$\begin{aligned} \Gamma, [a: A] &\preceq \Gamma, [a: \sim A] & \Gamma &\preceq \sigma(\Gamma) & \Gamma, a: ?_{\mathbf{m}} A &\preceq \Gamma \\ \Gamma, a: ?A &\preceq \Gamma, a: ?A, a: ?A & \Gamma, a: ?A &\preceq \Gamma, a: ?_{\star} A & \Gamma, a: !_{\star} A &\preceq \Gamma, a: !A \end{aligned}$$

The first rule identifies the type of a discharged occurrence and its dual, matching a type annotation which may be $(\nu a: A)$ or $(\nu a: \sim A)$. Then we have *exchange*, *weakening*, *contraction*. The last two axioms alter the mode: we can forget \star in $?_{\star} A$, and dually we can record it on $!A$.

³The reduction rule is not mentioned in [5], but the type rule is given and one of the authors relayed to me that reduction is assumed to be the same as in [17].

$$\begin{array}{c}
\text{(New)} \quad \frac{P \triangleright \Gamma, [a: A]}{(\nu a: A)P \triangleright \Gamma} \quad \text{(NewX)} \quad \frac{P \triangleright \Gamma, X \quad X \notin \text{ftv}(\Gamma)}{(\nu X)P \triangleright \Gamma} \quad \text{(Sub)} \quad \frac{\Gamma \preceq \Delta \quad P \triangleright \Delta}{P \triangleright \Gamma} \quad \text{(Str)} \quad \frac{P \equiv Q \quad Q \triangleright \Gamma}{P \triangleright \Gamma} \\
\text{(Ax)} \quad \frac{}{ab \triangleright a: A, b: \sim A} \quad \text{(Cut)} \quad \frac{P \triangleright \Gamma, a: A \quad Q \triangleright \Delta, a: \sim A}{P | Q \triangleright [a: A], \Gamma, \Delta} \quad \text{(OpenCut)} \quad \frac{P \triangleright \Gamma, a: !_m A \quad Q \triangleright \Delta, a: ?_m \sim A}{P | Q \triangleright \Gamma, \Delta, a: !_m A} \\
\text{(CoMix)} \quad \frac{P \triangleright \Gamma, \Theta \quad Q \triangleright \Delta, \Theta \quad \Theta \subseteq \{a: ?_* A\}}{P | Q \triangleright \Gamma, \Delta, \Theta} \quad \text{(1)} \quad \frac{}{\bar{a}() \triangleright a: \mathbf{1}} \quad \text{(\mathcal{L})} \quad \frac{}{a() | P \triangleright \Gamma, a: \mathcal{L}} \\
\text{(\otimes)} \quad \frac{P \triangleright \Gamma, b: A \quad Q \triangleright \Delta, c: B}{\bar{a}(b, c) | P | Q \triangleright [b: A, c: B], \Gamma, \Delta, a: A \otimes B} \quad \text{(\mathcal{R})} \quad \frac{P \triangleright \Gamma, b: A, c: B}{a(b, c) | P \triangleright [b: A, c: B], \Gamma, a: A \mathcal{R} B} \\
\text{(\oplus_1)} \quad \frac{P \triangleright \Gamma, b: A}{\bar{a} \triangleleft \mathbf{1}(b) | P \triangleright [b: A], \Gamma, a: A \oplus B} \quad \text{(\&)} \quad \frac{P \triangleright \Gamma, b: A \quad Q \triangleright \Gamma, c: B \quad b, c \notin \text{fn}(\Gamma)}{a \triangleright \{\mathbf{1}(b:A).P \parallel \mathbf{2}(c:B).Q\} \triangleright \Gamma, a: A \& B} \\
\text{(\forall)} \quad \frac{P \triangleright \Gamma, b: A \quad X \notin \text{ftv}(\Gamma)}{a(X, b) | P \triangleright [b: A], X, \Gamma, a: \forall X. A} \quad \text{(TyAl)} \quad \frac{P[A/X] \triangleright \Gamma}{X \mapsto A | P \triangleright \Gamma, X} \quad \text{(\exists)} \quad \frac{P \triangleright \Gamma, b: C \quad C = A[B/X]}{\bar{a}(B, b) | P \triangleright [b: C], \Gamma, a: \exists X. A} \\
\text{(!)} \quad \frac{P \triangleright ?\Gamma, \{x_i: A_i\}_{i \in I} \quad \forall i \in I. x_i \notin \text{fn}(?\Gamma) \quad I \neq \emptyset \quad \forall i \geq 2. \mathbf{m}_i = \star}{!a_i(x_i: A_i)_{i \in I}. P \triangleright ?\Gamma, \{a_i: !_m A_i\}_{i \in I}} \quad \text{(?D)} \quad \frac{P \triangleright \Gamma, b: A}{?\bar{a}(b) | P \triangleright [b: A], \Gamma, a: ?_m A}
\end{array}$$

Figure 1: Linear Logic Typing with Multiparty Promotion

Typing rules can be found in Fig. 1. We type modulo structure equivalence, a possibility suggested by [15] and used in [3]. This is because associativity of “ $|$ ” does not preserve typability, i.e., a cut between P and $(Q | R)$ may be untypable as $(P | Q) | R$; (νa) causes similar problems.

In (Cut) the name a is discharged and can then be closed with (New). (OpenCut) was added for two reasons. First, it is intuitive, since we are not required to close the name, i.e., to fix the number of clients of $!A$, departing from the de facto interpretation of “cut as composition under name restriction.” Second, it is needed for soundness. Take $R \doteq (\nu a: !A)(ab | ?\bar{a}(x) | P | !a(y).Q)$ typed with $R \triangleright \Gamma, b: !A$. Using (R-Ax) we obtain $R \longrightarrow ?\bar{b}(x) | P\{b/a\} | !b(y).Q$, which is *only* typable with the same interface by using (OpenCut); with (Cut) we obtain $\Gamma, [b: !A]$.⁴

Asynchronous messages can encode standard sessions (see [4, 5]): $\bar{a}(b, c)$ with type $A \otimes B$ maps to the session type $!_s \sim A.B$ or $!_s \sim B.A$. Dually, $a(x, y)$ with $\sim A \mathcal{R} \sim B$ maps to $?_s \sim A. \sim B$ or $?_s \sim B. \sim A$. To write processes in standard sessions style, with reuse of names (e.g., $\bar{a}b; a(x); \mathbf{0}$), we introduce abbreviations that use the second component for a ’s continuation:

$$\begin{array}{ll}
\bar{a}b; P \doteq (\nu x, y)(\bar{a}(x, y) | xb | P\{y/a\}) & a(x); P \doteq (\nu x, y)(a(x, y) | P\{y/a\}) \\
a \triangleleft l_k; P \doteq (\nu x)(\bar{a} \triangleleft k(x) | P\{x/a\}) & a \triangleright \{l_i.P_i\}_{i \in I} \doteq a \triangleright \{i(x_i).P_i\{x_i/a\}\}_{i \in I} \\
\bar{a}B; P \doteq (\nu x)(\bar{a}(B, x) | P\{x/a\}) & a(X); P \doteq (\nu X, x)(a(X, x) | P\{x/a\})
\end{array}$$

It is easy to check that linear redices commute with all other redices, and therefore a “real”

⁴Several works [17, 5, 20, 2] would not enjoy subject reduction if this example could be transferred: their cut rule requires $(\nu b)(\dots)$, which is here missing. These works don’t have “Mix” (here: (CoMix)), which we used in the example; but this should be checked, since “Mix” can be encoded with a new conclusion $c: \mathcal{L} \otimes \mathcal{L}$ [8, p. 100].

prefix would not have any effect on computation except to make it more sequential.

The rule (!) implements an extension of the logic:

$$\frac{? \Gamma, A}{? \Gamma, !A} \text{ (promotion)} \quad \textbf{becomes} \quad \frac{? \Gamma, A_1, \dots, A_n}{? \Gamma, !A_1, \dots, !A_n} \text{ (multi-promotion)}$$

Actually we need to employ some restrictions on this rule, which is why all conclusions except the first must have a \star -mode. Since there is no contraction for $?_\star$,⁵ all the $a_i: ?_\star \sim A_i$ ($i \geq 2$) will come from terms with just one call to the session.⁶ The first conclusion, $a_1: !_{\mathbf{m}_1} A_1$, can have standard mode ($\mathbf{m}_1 = \varepsilon$), which allows a client's call with $a_1: ? \sim A_1$ to be connected to (i.e., to depend on) other calls on a_1 . In this way we provide a *hook* for one client to participate in another instance of the same session, and this facilitates a form of *dynamic join*. We return to this concept in the first example.

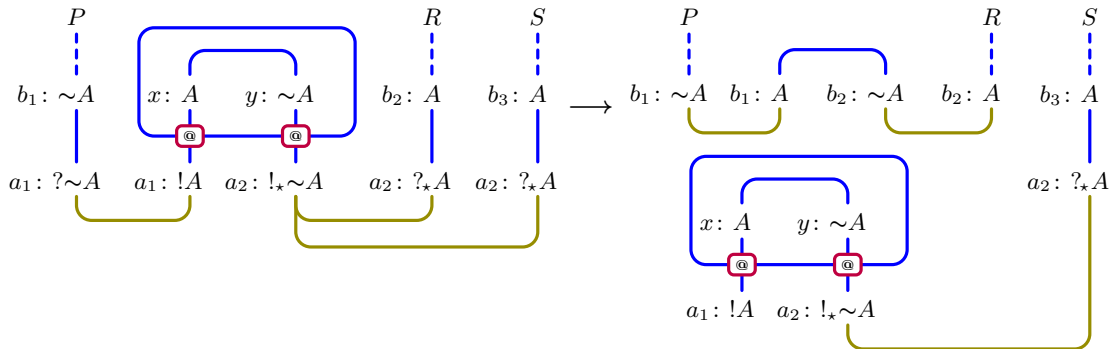
Finally, the sidecondition in $(\& !)$ forbids premises from having multiple copies of a name (e.g., $x: ?A, x: ?A$) which should be removed in the conclusion; essentially it forces contractions (by \prec). Other rules are immune by the well-formedness of $\Gamma, [a: A]$.⁷

Expressiveness & Properties The system is an extension of proof nets, in process form, so it can encode System F, inductive sessions (using second-order features), etc. Due to space limitations we only show two examples: (a) how shared channels can be simulated with synchronisation; (b) the (two Buyer, one Seller) protocol from [11].

a) channels Non-determinism can be expressed by sharing a channel between multiple competing processes trying to send and receive messages. This is impossible with existing logical sessions systems, and more generally if we follow the logic “by the book.” A channel a with i/o type $(A, \sim A)$, i.e., that exports two complementary capabilities A and $\sim A$, can be encoded by the two names a_1 and a_2 in $!a_1(x:A) a_2(y:\sim A).xy \triangleright a_1: !A, a_2: !_\star \sim A$. The channel is used by terms with $a_1: ? \sim A$ or $a_2: ?_\star A$, and there can be multiple instances of each, giving rise to critical (non-deterministic) pairs. Moreover, A can be linear, i.e., we can communicate linear values through shared channels, which is a novel feature. For example:

$$\begin{aligned} & ?\bar{a}_1(b_1) \mid ?\bar{a}_2(b_2) \mid ?\bar{a}_2(b_3) \mid !a_1(x:A) a_2(y:\sim A).xy \mid P \mid R \mid S \\ & \xrightarrow{(a)} b_1 b_2 \mid ?\bar{a}_2(b_3) \mid !a_1(x:A) a_2(y:\sim A).xy \mid P \mid R \mid S \\ & \xrightarrow{(b)} b_1 b_3 \mid ?\bar{a}_2(b_2) \mid !a_1(x:A) a_2(y:\sim A).xy \mid P \mid R \mid S \end{aligned}$$

First, note that confluence is lost: assume P, R, S cannot reduce and it becomes obvious. The graphical notation with a reduction of the first possibility is depicted below.



⁵More accurately: contraction of $?_\star \sim A_i$ is *multiplicative*.

⁶In the sense that two calls can never depend on each other.

⁷It is subtle but due to the variable convention, $(\&)$ is actually immune too; the condition serves for clarity.

It is possible that P has another call to a_1 , but by the restriction on $?_*$ -types there cannot be a “trip” from b_2 to a_2 , as this would lead to a cycle. Concretely, if R has another call to a_2 , then it is from a part disconnected to b_2 , and similarly for S ; see (CoMix).⁸

b) multiparty interactions The (two Buyer, one Seller) protocol from [11] is shown below, with insignificant adaptations, using the previously explained abbreviations (we omit some signals for $\mathbf{1}/\mathcal{L}$):

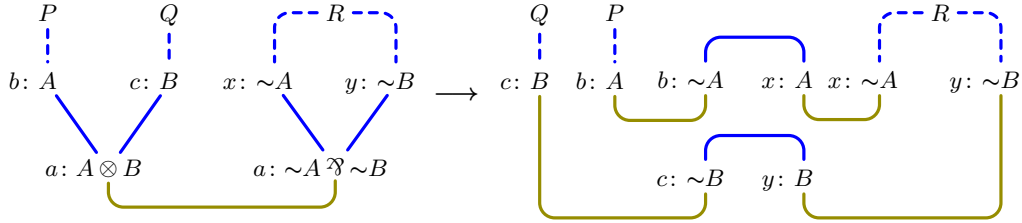
$$\begin{aligned} \text{Buyer1} &\doteq (\nu b_1) (? \bar{a}_1(b_1) \mid \bar{b}_1 \text{“The Art of War”}; b_1(\text{quote}); (\nu z)(\bar{b}_1 z; \mathbf{0} \mid \bar{z} \text{quote}/2; P_1)) \\ \text{Buyer2} &\doteq (\nu b_2) (? \bar{a}_2(b_2) \mid b_2(\text{quote}); b_2(z); z(\text{contrib}); b_2 \triangleleft \text{ok}; \bar{b}_2 \text{“SW12 3AZ”}; b_2(\text{date}); P_2) \\ \text{Seller} &\doteq !a_1(x_1) a_2(x_2). \left(\begin{array}{l} x_1(\text{title}); \bar{x}_1 20 \text{€}; \bar{x}_2 20 \text{€}; x_1(z); \bar{x}_2 z; \\ x_2 \triangleright \{\text{ok}.x_2(\text{address}); \bar{x}_2 \text{“7/Feb”}; Q \parallel \text{quit.}\mathbf{0}\} \end{array} \right) \end{aligned}$$

We note that the simplicity of the example has not been sacrificed, compared to the code in [11]. One difference is that we passed z from Buyer1 to Buyer2 *through* Seller using $x_1(z); \bar{x}_2 z$, when in [11] all names are known to all participants. We do not employ the *global types* of [11], but there is a *proof net* for Buyer1 \mid Buyer2 \mid Seller, not shown due to space constraints, and we postulate that:

The proof net can serve as an alternative notion of global type.

Outline of results The expected soundness result for reduction, $P \triangleright \Gamma$ and $P \longrightarrow P'$ implies $P' \triangleright \Gamma$, is obtained in a standard way, but fails without the \star -mode. Strong Normalisation (**sN**), i.e., $P \triangleright \Gamma$ implies that *all* reduction sequences from P are finite, is shown by an adaptation of the reducibility candidates technique from [8]. The loss of confluence complicates the proof, which is in fact obtained for an extended (confluent) reduction relation using a technique of [6], from which we derive as a corollary the result. For **sN** we prove the (initially) stronger property of *reducibility* [8], which can also serve as a very strong progress guarantee.

A Curry-Howard correspondence can be obtained easily for a fragment of the language. For the multiplicative, additive, and second-order cut-elimination we only need to perform extra axiom cuts (i.e., substitutions). For exponentials, we restrict replications to a single input and simulate the actual copying (with contraction links) that takes place in proof nets with sharing and sequentialised cut-elimination steps. Indeed, there is still a loss of parallelism compared to standard proof nets, but the term language is more realistic. We show just one case of cut-elimination, the cut $(\otimes - \wp)$, implemented by $\bar{a}(b, c) \mid a(x, y) \longrightarrow bx \mid cy$, adding appropriate contexts (P, Q, R) :



3 Conclusion

We claim that our language is simpler and proof-theoretically more appealing than related works such as [3]: structured interactions take place as expected (*fidelity*), but parallelism is not inhibited by the use of prefix, which cannot anyway alter the result in a deterministic setting. It is really a question of proof nets vs. sequent proofs, and in logic the first are almost always preferable. Even with synchronisation and the induced non-determinism, the system we propose retains good properties, for example it seems to be the first approach to multiparty behaviours that enjoys strong normalisation. Finally, our notion of *proof net as global type* seems to be a reasonable solution for logically founded multiparty sessions.

⁸In general, derelictions can be connected; try with $a: ?(A \oplus \sim A)$.

In relation to Abramsky’s interpretation [1], it is close to proof nets *with boxes*, i.e., to a completely synchronous calculus. Moreover, it is not so friendly syntactically, it does not have a notion of bound name, copying of exponentials is explicit (no *sharing*), and of course it is completely deterministic.

An interesting future direction would be to obtain a *light* variation of our system, *e.g.*, following [9]. Then we could speak of implicit complexity for multiparty sessions, similarly to what has been done in [13] for binary sessions. Due to space restrictions, more examples and all proofs have been omitted. These will appear in a longer version, see <http://www.di.fc.ul.pt/~dimitris/>.

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