1 Introduction

Logical duality has been a key issue in the study of natural language and, in particular, in the study of quantification as this happens be expressed in natural language. It is a pattern noticed in the semantics of many linguistic expressions and phenomena, ranging from the realm of determiners to the realm of temporality and modality, including topics such as the semantics of the adverbials still/already or of the conjunctions because/although, etc. (Löbner, 1987, ter Meulen, 1988, König, 1991, Smessaert, 1997).

Under this pattern, one recurrently finds groups of syntactically related expressions whose formal semantics can be rendered as one of the operators arranged in a square of duality. Such a square is made of operators that are interdefinable by means of the relations of outer negation, inner negation, or duality (i.e. concomitant outer and inner negation):

(1)

The semantic values of the English expressions every $N$, no $N$, some $N$ and not every $N$, or their translational equivalents in other natural languages, provide the classical example of an instantiation of the square above:

In this connection, Löbner (1987) suggested that the emergence of a — notoriously non trivial — square of logical duality between the semantic values of natural language expressions is a major empirical touchstone to ascertain their quantificational nature; and van Benthem (1991), while noting that the ubiquity of the square of duality may be the sign of a semantic invariant possibly rooted in some cognitive universal, highlighted its heuristic value for research on quantification inasmuch as "it suggests a systematic point of view from which to search for comparative facts" (p.23)— a hint we seek to explore in this paper.

Given our purpose here, it is of note that the square of duality in (1) is different and logically independent from the classical square of oppositions in (2):

(2)
The difference lies in the fact that duality, inner negation and outer negation are third order concepts, while compatibility, contrariness and implication are second order concepts. As a consequence, it is possible to find instantiations of the square of oppositions without a corresponding square of duality, and vice-versa (vd. Löbner, 1987:56 for examples and discussion).

Although the two squares are logically independent, the empirical emergence of a square of oppositions for the semantic values of natural language expressions naturally raises the question about the possible existence of an associated square of duality, and about their quantificational nature. This is where we get focused into the research issue of this article, given the emergence of a square of oppositions with the grammatical constraints on anaphoric binding, also known as binding principles.

2 Anaphoric binding constraints

Binding principles capture generalisations concerning the relative positioning of anaphors with respect to their admissible antecedents in the grammatical geometry of sentences. From an empirical perspective, these constraints stem from what appears as quite cogent generalisations and exhibit a universal character, given the hypothesis of their parameterised validity across natural languages (Dalrymple, 1993). From a conceptual point of view, in turn, the relations among the definitions of binding constraints involve non-trivial cross symmetry, which lends them a modular nature and provides further strength to the plausibility of their universal character. Binding principles have thus been considered one of the most significant modules of grammatical knowledge, usually termed as binding theory in theoretical linguistics.

We follow here the definition proposed in (Pollard and Sag, 1994) for these constraints, and subsequent extension in (Xue et al., 1994, Branco and Marrafa, 1999), which are presented below, together with some illustrative examples. These constraints on the anaphoric capacity of nominals induce a partition of the set of these anaphors into four classes, which are then identified as being one of the following four anaphoric types: short-distance reflexive, pronoun, long-distance reflexive, and non-pronoun.

Principle A: A locally o-commanded short-distance reflexive must be locally o-bound.
[Lee’s friend]j thinks [[Max’s neighbour], likes himselfijk].

Principle Z: An o-commanded long-distance reflexive must be o-bound.
[O amigo do Lee]jacha [que [o vizinho do Max]l gosta dele próprioijk]. (Portuguese)
the friend of the Lee thinks that the neighbour of the Max likes of_him self
[Lee’s, friend]j, thinks [[Max’s neighbour], likes himijk/himself].

Principle B: A pronoun must be locally o-free.
[Lee’s friend]j, thinks [[Max’s neighbour], likes himijk].

Principle C: A non-pronoun must be o-free.
[Lee’s friend]j, thinks [[Max’s neighbour], likes the boyijk].

The definitions of binding constraints are built in terms of a few auxiliary notions. The notion of o-binding is such that x o-binds y iff x o-commands y and x and y are coindexed, where
coindexing is meant to represent anaphoric links.\(^1\)

\textit{O-command} is a partial order under which, in a clause, the Subject o-commands the Direct Object, the Direct Object o-commands the Indirect Object, and so on, following the usual obliqueness hierarchy of grammatical functions; in a multiclausal sentence, the upward arguments o-command the successively embedded arguments.\(^2\)

The notion of \textit{local domain} for an anaphoric nominal \(n\) involves the partition of sentences and associated grammatical geometry into two zones of greater or less proximity with respect to \(n\). Typically, the local domain coincides with the selectional domain of the predicator subcategorising the anaphor. In some cases, there may be additional requirements that the local domain is circumscribed by the first upward predicator that happens to be finite, bears tense or indicative features, etc.\(^3\)

Given this introductory remarks on anaphoric binding constraints, the key observation to make now with respect to the generalisations in (3) above is that, when stripped away from procedural phrasing and non-exemption requirements,\(^4\) they instantiate the following square of oppositions:

There are two pairs of \textit{contradictory} constraints, which are formed by the two diagonals, (A, B) and (C, Z). One pair of \textit{contrary} constraints (they can be both false but cannot be both true) is given by the upper horizontal edge (A, C). One pair of \textit{compatible} constraints (they can be both true but cannot be both false) is given by the lower horizontal edge (Z, B). Finally two pairs of subcontrary constraints (the first coordinate implies the second, but not vice-versa) are obtained by the vertical edges, (A, Z) and (C, B).

Given this new square of oppositions, the natural question to ask is whether this is a sign that binding principles are the visible effect of some underlying quantificational structure.

\section*{3 Phase quantification}

Our major point here is to argue that this question can be answered affirmatively. In the light of the considerations in Section 1, we proceed by showing that there is a square of duality associated with the grammatical constraints on anaphoric binding. But before this result may be fully worked out, some analytical tools are to be introduced first.

We resort to the notion of phase quantification, introduced in (Löbner, 1987) to study the semantics of aspectual adverbials and shown to be extended to characterise quantification in general. For the sake of concreteness, consider a diagrammatic display of the semantics of such adverbials:

---

\(^1\) There are anaphors that are subject-oriented, in the sense that they only take antecedents that have the grammatical function Subject. Some authors (e.g. Dalrymple, 1993) assume that this should be seen as an intrinsic parameter of binding constraints and aim at integrating it into their definition. In this point we follow previous results of ours reported in (Branco, 1996), where the subject-orientedness of anaphors is argued to be, not an intrinsic feature of binding constraints, but one of the surfacing effects resulting from the non linear obliqueness hierarchy associated with some predicates (or with all of them in some languages).

\(^2\) The o-command relation is defined on the basis of obliqueness hierarchies successively embedded along the relation of subcategorization: "Y o-commands Z just in case either Y is less oblique than Z; or Y o-commands some X that subcategorises for Z; or Y o-commands some X that is a projection of Z." (Pollard and Sag, 1994:279). For a discussion of the empirical justification for obliqueness hierarchies as well as references on this topic, see (Pollard and Sag, 1987:Sec.5.2).

\(^3\) Vd. (Dalrymple, 1993) for details.

\(^4\) For a detailed discussion of exemption occurrences of reflexives see next footnote.
Very briefly, phase quantification requires the following ingredients: (i) an order over the domain of quantification; (ii) a parameter point $t$; (iii) a property $P$ defining a positive semiface in a sequence of two opposite semiphases; and (iv) the starting point of a given semiface.

For the analysis of aspectual adverbials in terms of phase quantification, the order of (i) is the time axis; the parameter point $t$ of (ii) is the reference time of the utterance; the relevant property $P$ of (iii) denotes the instants where the proposition containing the adverbial holds (with the adverbials no longer and still bearing the presupposition that $\sim P$ precedes $P$, and not yet and already bearing the presupposition that $\sim P$ precedes $P$); and the starting point in (iv) is $I(R,t)$, the infimum of the set of the closest predecessors of $t$ which form an uninterrupted sequence in phase $R$ (vd. Löbner, 1987, 1989) for a thorough definition.

Given these correspondences, the aspectual adverbials can be analysed as expressing the following quantifiers:

\[
\begin{align*}
\text{still'}: & \quad \lambda P. \text{every'}(\lambda x.(\text{Inf}(P,t) < x \leq t), P) \\
\text{not_yet':} & \quad \lambda P. \text{no'}(\lambda x.(\text{Inf}(\sim P,t) < x \leq t), P) \\
\text{no_longer'}: & \quad \lambda P. \text{not_every'}(\lambda x.(\text{Inf}(P,t) < x \leq t), P) \\
\text{already':} & \quad \lambda P. \text{some'}(\lambda x.(\text{Inf}(\sim P,t) < x \leq t), P)
\end{align*}
\]

4 The quantificational force of anaphors

With this in place, the empirical generalisations captured in the definition of binding principles in (3) can be argued to be the visible effect of the phase quantificational nature of the corresponding nominals. In this section we will show how anaphoric nominals can be analysed as expressing one of four quantifiers acting on the domain of reference markers arranged in terms of the grammatical obliqueness order of their clauses.

4.1 Phase quantification ingredients

Phase quantification here is assumed to unfold not over entities of the extra-linguistic universe, but over entities in the universe of grammatical representations, vz. reference markers a la Kamp. Its ingredients are set up as follows:

(i) **Order**: reference markers are ordered according to the o-command relation;
(ii) **Parameter point**: $t$ is set up as $a$, the reference marker of the antecedent of the anaphoric nominal at stake;
(iii) **Phase property**: $P$ is set up as $D$, denoting the set of markers in the local grammatical domain of the anaphor.

For an anaphoric nominal $N$ in a given sentence $s$, $D$ is thus determined by the position of $N$ in the obliqueness order which $N$ enters in $s$. Given $m$, the reference marker of $N$, semiface $D_m$ is a stretch containing $m$ and the markers that are less than $m$ in the obliqueness order. In case $\sim D_m$ is presupposed to precede $D_m$, the first predecessor in $D_m$ is local with respect to $m$; and in case semiface $D_m$ is presupposed to precede $\sim D_m$, $D_m$ is such that the last successor in it (it itself) is local with respect to $m$. Phase $D_m$ is such that the closest $D_m$ neighbour of semiface $\sim D_m$ is local with respect to $m$.

Given this new ingredients for phase quantification and with the appropriate replacements in the square in (4), one gets four phase quantifiers — we termed $Q_Z$, $Q_B$, $Q_c$ and $Q_A$ — entering the square of duality and aligning with other quantifiers of similar quantificational force at each of

\[
\begin{align*}
\text{still'}: & \quad \lambda P. \text{every'}(\lambda x.(\text{Inf}(P,t) < x \leq t), P) \\
\text{not_yet':} & \quad \lambda P. \text{no'}(\lambda x.(\text{Inf}(\sim P,t) < x \leq t), P) \\
\text{no_longer'}: & \quad \lambda P. \text{not_every'}(\lambda x.(\text{Inf}(P,t) < x \leq t), P) \\
\text{already':} & \quad \lambda P. \text{some'}(\lambda x.(\text{Inf}(\sim P,t) < x \leq t), P)
\end{align*}
\]
As we will check now in the subsections below, these four phase quantifiers ensure the same empirical predictions as secured by the four binding principles stated in (3).

4.2 Short-distance reflexives
The quantifier expressed by short-distance reflexives is associated with the presupposition that \( \sim D.D \). It receives the following definition:

\[
Q_A : \lambda P. \text{some}'(\lambda x.(I(\sim P,a) < x \leq a,P))
\]

This is easily interpreted against the diagram corresponding to the example sentence *Kim said Lee thinks Max hit himself*[^1]. In the diagram, \( k, l, m \) and \( h \) stand, respectively, for the reference markers of *Kim*, *Lee*, *Max* and *himself*; and \( x_1, \ldots, x_n \) stand for the markers not in the obliqueness relation of \( h \), including those possibly introduced in other sentences of the discourse or available in the context (Hasse diagrams are displayed with a turn of 90º right):

\[
Q_A(D_h) \text{ is satisfied iff between the bottom of } \sim D_h \text{ most close to the antecedent } a \text{ and } a \text{ inclusive, there is at least one reference marker in } D_h. \text{ As } \sim D_h \text{ precedes } D_h, \text{ this amounts to requiring that } a \text{ be in } D_h, \text{ the local domain of } h \text{ here, and consequently that } a \text{ be a local o-commander of } h, \text{ which matches the requirement in Principle A. Binding phase quantifier } Q_A \text{ shows positive existential force and short-distance reflexives align in the square of duality with items like } \text{some } N, \text{ already, possibly, etc.}
\]

4.3 Pronouns
The phase quantifier expressed by pronouns, in turn, lies at the same corner as the quantifiers no'(R) or not_yet' in (5):

\[
Q_B : \lambda P. \text{no}'(\lambda x.(I(\sim P,a) < x \leq a,P))
\]

The presupposition conveyed by these anaphors is also that \( \sim D.D \), and \( Q_B \) is easily understood when considering the diagrammatic description of an example like *Kim said Lee i thinks [Max hit him]*:

\[
Q_B(D_h) \text{ is satisfied iff no reference marker between the bottom of } \sim D \text{ and the antecedent } a \text{ inclusive is in } D, \text{ which implies that } a \text{ has to be in } \sim D, \text{ i.e. it has to be outside the local domain of the pronoun, as in the generalisation captured by Principle B. This includes not only the markers } k \text{ and } l \text{ of } Kim \text{ and Lee, in the upwards clauses, but also } x_1, \ldots, x_n \text{ thus allowing for non-sentential anaphoric links for the pronoun (including those that are}
\]
discourse- or context-driven, deictic, etc.).

### 4.4 Long-distance reflexives

Turning to long-distance reflexives, we consider the working example from Portuguese *O amigo de Kim disse que ele próprio acha [que Lee viu Max].* / *Kim’s friend, said “ele próprio”, thinks [Lee saw Max].* Its diagrammatic display can be depicted as follows:

![Diagram](https://example.com/diagram.png)

The phase quantifier expressed by long-distance reflexives has found at the same corner as the quantifiers every(R) or still in (5):

\[
Q_Z: \lambda P.\text{every}'(\lambda x.(P,a) < x \leq a,P)
\]

As with short-distance reflexives, the antecedent \(a\) is here required to occur in \(D\), though the presupposition conveyed now is that the positive semiphase \(D\) is followed by the negative semiphase \(\sim D\). Taking into account the definition of \(D\) above, the antecedent is thus required to be an o-commander – local or not – of the marker \(e\) of the anaphoric nominal.

The semantics of the corresponding phase quantifier is such that, for \(Q_Z(D_e)\) to be satisfied, between the bottom of the uninterrupted sequence \(D_e\) closest to an admissible antecedent \(a\) and \(a\) inclusive, every reference marker is in \(D_e\). This amounts to requiring \(a\) to be in \(D_e\), i.e. to imposing that any admissible antecedent is an o-commander of the long-distance reflexive, as required by Principle Z.\(^5\)

### 4.5 Non-pronouns

The quantifier expressed by non-pronouns appears at the same corner as quantifiers like not_every'(R), no_longer', etc.:

\[
Q_C: \lambda P.\text{not_every}'(\lambda x.(P,a) < x \leq a,P)
\]

Let us consider a first version of the diagram for the example [Kim’s friend] said the boy.

---

\(^5\) When reflexives occur in a syntactic position where they have no possible antecedent o-commanding them in their domain, their anaphoric capacity is exempt from the usual discipline and they present a so-called logophoric behaviour. This is illustrated in the following example from Golde (1999:73), where *herself* picks an antecedent outside its binding domain, the NP *the portrait of herself*: Mary, thought the artist had done a bad job, and was sorry that her parents came all the way to Columbus just to see [the portrait of herself].

Under the quantificational analysis of binding constraints, to a reflexive \(M\) in an exempt position (i.e. in the bottom of \(D\)), there corresponds the maximum “shrink” of \(D\), as this is the singleton whose sole element is \(m\). This maximum shrink has a disturbing impact only in the phase quantifiers for which the antecedent \(a\) is to be found in \(D\), namely \(Q_A\) and \(Q_Z\). In these cases, for \(a\) to be in \(D\) and the relevant quantification to be satisfied, \(a\) can only be identified with \(m\) itself.

As \(m\) is engaged in this anaphoric anchoring loop, its non vacuous interpretation remains to be accomplished. Admittedly, an overarching interpretability requirement is in force in natural languages ensuring the “meaningful” anchoring of anaphors: For an exempt reflexive to be non vacuously interpreted, an antecedent — inevitably outside its binding domain now — has to be fixed. Logophoricity appears thus as an exceptional anaphoric behaviour of reflexives that shows up when their interpretation has to be untied from anchoring loops formed by virtue of their markers occurring in the bottom of the positive semiphase \(D\).
An admissible antecedent $a$ of $b$ should be required to occur in $\sim D_b$, which implies that $a$ cannot be an o-commander of $b$, thus rendering the same constraint as expressed by Principle C.

As in previous diagrams, the negative semiphase $\sim D$ is taken here as the complement set of $D$. Correct empirical prediction requires however this assumption to be refined and a more accurate definition of $\sim D$ be provided for phase quantification in non-linear orders — as the one under consideration — where not all elements of the quantification domain are comparable.

For $Q_C(D_b)$ to be satisfied, between the bottom of $D_b$ and the antecedent $a$ inclusive, not every reference marker is in $D_b$. In examples as the one above, the denotation of $\lambda x.(I(D_b,a)<x \leq a)$, the restrictor of $Q_C$, is always empty: It is not the case that $I(D_b,a) \leq a$ because when $a=k$ (or $a=x_i$ for any $i$), $a$ is not comparable to any element of $D_b$, including its bottom. Hence, $\neg \text{every} (\lambda x.(I(D_b,a)<x \leq a), D_b)$ is false whatever reference marker $k$ or $x_i$ happens to be taken as the antecedent for $b$. As a consequence, the specific anaphor resolution in the example above would be incorrectly ruled out.

This suggests that when phase quantification operates on non-linear orders, negation of the positive phase $P$ may be slightly more sophisticated than simple Boolean negation rendering its complement set. We are taught that negation of $P$ also involves the lifting of the complement set, $\bar{P}_{\bot}$, with $\neg$ equal to the top of $P$ ($b$ in the working example above) when occurs the presupposition that $\neg P$.

With this fine-tuned definition of the negative semiphase, the diagrammatic display for our working example becomes:

This specification of $\neg P$ correctly ensures that $Q_C(D)$ is satisfied iff any of the grammatically admissible anaphoric links specified in Principle C holds between non pronouns and their antecedents.

---

6 For the sake of formal uniformity, when $\neg P.P$, the order-theoretic dual of this definition for $\neg P$ can also be assumed.
5 Discussion: quantification and reference

5.1 Quantification

Many authors have stressed that there is no correspondence between surface and logical form of quantificational expressions of natural languages. Löbner emphasised this non-correspondence by pointed out that, while domain restrictor and quantified predicate are rendered by two different surface expressions in nominal quantification, only the quantified predicate is superficially available in phase quantification as this is expressed by aspectual adverbials. With phase quantification expressed by anaphors, this gulf between surface and logical form has widened further: There is no surface expression either for the domain restrictor of quantification or for the quantified predicate.

Other important implications for our understanding of the realm of quantification in natural languages might have been uncovered as well by the results presented here. Quantification is extended to universes which are possibly non-linearly ordered, and more important, to universes whose elements are not entities of the “extra-grammatical” universe, but entities of the “intra-grammatical” world itself.

The models against which binding phase quantification is to be interpreted are not representations of the world, with everyday entities like donkeys, farmers, etc., but grammatical representations, with entities like reference markers, grammatical functions, etc.. Hence, satisfaction of a formula made out of a binding phase quantifier turns out to be a well-formedness constraint on the syntax and semantics of the sentence where the corresponding anaphor occurs: For the meaning of “classic” quantification to be determined, one has to know how the world has to be for it to be true; for the meaning of binding phase quantification to be determined, one has to know how the grammatical representation has to be for it to be true.

5.2 Symmetric dualities

Finally, it is worth considering the implications of the results reported above for the overall semantic make up of nominals.

It is a shared wisdom that nominals convey either quantificational or referential force, and a large bulk of the research on the semantics of nominals has been concerned with determining which side of this duality definite descriptions belongs to. For the sake of the argument, let us assume that definites are referential terms. Let us also embrace the result that proper nouns are ruled by binding principle C.

Given these assumptions, the analysis developed in this article imply that nominals with “primary” referential force (he, the book, John,...) have a somewhat “secondary” quantificational force: They express quantificational requirements — over reference markers in grammatical representations —, but cannot be used to directly quantify over extra-linguistic world entities as the other “primarily” quantificational nominals (every man, most students,...) do.

This, however, turns out not to be that much surprising if one observes a symmetric duality with regards quantificational nominals, apparent when they act as antecedents in e-type anaphora, as in Few students, came to the party but they, had a good time. The analysis of e-type anaphora proposed by some authors can be seen as implying that nominals with “primary” quantificational force have a somewhat “secondary” referential force: These nominals have enough referential strength to introduce reference markers in the grammatical representation that can be picked as antecedents by anaphors — and thus support the referential force of the latter —, but they cannot be used to directly refer to extra-linguistic entities as the other “primarily” referential terms do.7

If the results reported here are philosophically meaningful, the duality quantificational vs. referential nominals is less strict but more articulated than supposed before. Every quantificational or referential nominal (possibly taking indefinite descriptions aside) makes a dual contribution in both dimensions of quantification and reference, but with respect to different universes: “Primarily” referential nominals have a dual semantic nature that is symmetric of the

7 We have in mind here analysis of e-type anaphora like the one proposed by Kamp and Reyle, 1993, an in particular their use of Σ-abstraction in Chap.4.
dual semantic nature of “primarily” quantificational ones.

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